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Mathematical model of thermal breakdown of a plane layer of a polar dielectric

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Abstract. For a plane layer of a polar dielectric, which has a nonmonotonic dependence of the dielectric losses on temperature, a mathematical model describing the temperature state of this layer before its thermal breakdown is constructed. On the basis of this model, the connection between the breakdown voltage and the parameters determining the properties of the dielectric and the temperature distribution in the layer preceding the onset of thermal breakdown is established in an integral form. A quantitative assay of the model is carried out for the two types of polymer materials used as dielectrics.

Mathematics Subject Classification. Primary 97M50; Secondary 74A35.

Keywords. Polar dielectric, Thermal breakdown, Mathematical model, Temperature state.

1. Introduction

Polymeric dielectric materials are widely used in various electrical and electronic devices working at high operating parameters. Dielectric losses in such materials lead to a significant intensity of energy release that causes the increase in the working temperature of the dielectric, which can result in the growth of these losses. With the limited environment heat removal of the energy released in the dielectric, a positive feedback occurs, leading to a further temperature increase. As a result, the steady-state temperature of the dielectric may not be possible, the dielectric temperature will exceed the allowable value for the polymer material used and there appears a so-called thermal dielectric breakdown (in contrast to the electric breakdown [1,2]. Thermal breakdown results in thermal destruction of the material (by melting, carburization, and charring).

Among polymeric materials, there exist nonpolar and polar dielectrics differing in the nature of the dependence of the dielectric loss on temperature. For the nonpolar dielectrics, the dielectric losses monotonically increase as the temperature increases and are associated mainly with the effect of polymer reach-through conductivity effect. In polar dielectrics, along with reach-through conductivity, an important role is played by the polarization of separate fragments of the polymer molecular structure, which leads to a nonmonotonic temperature dependence of the dielectric loss, having one or more maxima [3–6] in the operating temperature range. With the operating voltage frequency increase, these maxima tend to shift toward higher temperatures.

The well-known theories of dielectrics thermal breakdown have been developed for a monotonically increasing temperature dependence of dielectric losses [7,8]. For nonpolar polymer dielectrics, this dependence is usually approximated by an exponential function with an exponent proportional to the temperature. To expand the possibilities for a quantitative assay of the conditions for the thermal breakdown occurrence, modern methods of mathematical modeling can be used [9,10]. In this paper, for a plane layer of the polar polymer dielectric, the thermal breakdown model is constructed in case of such arbitrary dependence, including a nonmonotonic one or more maxima. In order to establish the effect of the properties of such dielectric on the thermal breakdown occurrence, a quantitative assay of this model was carried out under simple heat exchange conditions on the surfaces of a plane layer with the surrounding medium.

2. Basic relations

The volume power of energy release, caused by the dielectric loss, is determined by the ratio [11,12]

$$q_V = E^2\gamma, \tag{2.1}$$

where E is the electric field intensity, measured in V/m; γ is the dielectric per-unit-volume direct current conductance under the AC voltage, measured in $1/(\Omega\ \mathrm{m})$ and is equal to

$$\gamma = \omega\varepsilon_0\varepsilon' \,\mathrm{tg}\delta = 2\pi f_0\varepsilon_0\varepsilon_r \,\mathrm{tg}\delta \approx f_0\frac{\varepsilon_r \,\mathrm{tg}\delta}{1.8\times 10^{10}}. \tag{2.2}$$

Here, ω is the angular (circular) frequency of voltage variation, rad/s; $f_0 = \omega/(2\pi)$ is the AC voltage frequency, Hz; $\varepsilon_0 \approx 8.8542\times 10^{-12}$ is the electric constant, F/m; ε' is the relative permittivity of a dielectric; and δ is the dielectric loss angle.

The main contribution to the γ change with temperature T is introduced by the dependence on T of the tangent of the dielectric loss angle tgδ, whereas the effect of the variation ε' with the temperature is relatively small, and when considering the dielectric thermal breakdown, it is usually assumed that $\varepsilon' = \mathrm{const}$. In this case, in the plane dielectric layer, the electric field is constant and equal to

$$E = \Delta U/h, \tag{2.3}$$

where ΔU is the absolute value of the electrical potential difference on the surfaces of the dielectric layer, and h is the thickness of this layer. Hence, the relation (2.1), with account for formulae (2.2) and (2.3), can be represented as

$$q_V(T) = \frac{(\Delta U)^2}{1.8\times 10^{10}h^2} f_0\varepsilon' F_*(T). \tag{2.4}$$

Here, $F_*(T)$ is the dimensionless temperature function describing the dependence tgδ on T. Figure 1, as an example, gives the graphs of this function for various values of f_0 for polyvinyl acetate [13], having a single maximum in the operating temperature range. For alicyclic polyimide [14], the graphs of this function at a frequency of 1 kHz have several maximum (Fig. 2), and this heat-resistant polymer remains operable as a dielectric up to the temperature of approximately 350 °C.

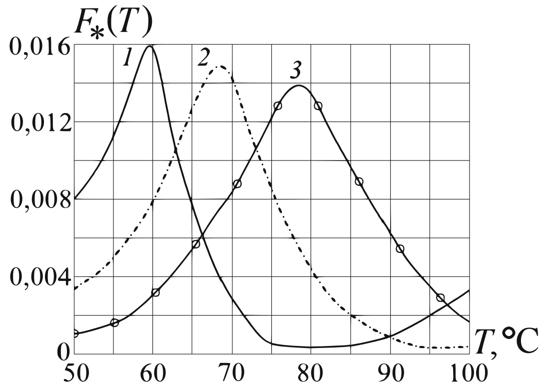

FIG. 1. The graphs of the function $F_*(T)$ for polyvinyl acetate for various values of the frequency f_0: 1 — 50 Hz; 2 — 1 kHz; 3 — 10 kHz

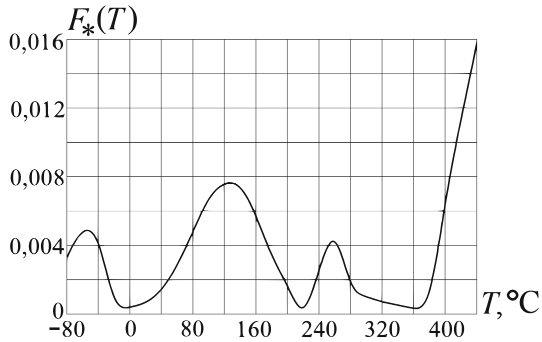

FIG. 2. The graphs of the function $F_*(T)$ for alicyclic polyimide for the frequency $f_0 = 1$ kHz

3. Formulation of the problem

If the thickness h is small as compared with the dimensions of the dielectric layer in tangential directions, and the heat transfer conditions on each of the surfaces of this layer are constant, then the steady temperature distribution $T(z)$ can be considered as one-dimensional, depending only on one coordinate z in direction of the common normal to these surfaces. Counting of this coordinate is selected on one of the surfaces of the layer. Then, the coordinate of the points on the other surface is equal to $z = h$.

The steady-state temperature distribution in a plane dielectric layer, whose material has a thermal conductivity λ, must satisfy the differential equation [15]

$$\lambda \frac{d^2T(z)}{dz^2} + q_V(T) = 0. \tag{3.1}$$

To solve this equation, it is necessary to formulate the boundary conditions on the surfaces of the layer. Let us choose these conditions in the following form

$$\left.\frac{dT(z)}{dz}\right|_{z=0} = 0, \quad T(h) = T_*, \tag{3.2}$$

where T_* is the set temperature on the layer surface at $z = h$. The first equality (3.2) corresponds to condition for an ideal thermal insulation of the layer surface at $z = 0$, which is equivalent to considering a layer with a thickness of $2h$, on both surfaces of which the same temperature values are given.

The problem, which is determined by the nonlinear differential equation (3.1) and the boundary conditions (3.2), can have several solutions, but it is also possible that there is no solution to this problem. The latter means impossibility of the existence of the steady-state temperature distribution in the dielectric layer that characterizes the thermal breakdown state of this layer. To establish a combination of the determining parameters for which the problem (3.1), (3.2) does not have a solution, it is expedient to proceed to a ratio which in an integrated form connects temperature distribution in the dielectric layer with these parameters. Using the well-known procedure for reducing the derivative order in (3.1), similar applied at integration of the movement equation in analytic mechanics [16], we write

$$\zeta = 1 - \frac{1}{\beta^2 T_*^{1/2}} \int\limits_{T(\zeta)}^{T_0} \left(2 \int\limits_{T'}^{T_0} F_*(T'')\,\mathrm{d}T'' \right)^{-1/2} \mathrm{d}T', \tag{3.3}$$

where $\zeta = z/h$, $\beta = (\Delta U)^2 f_0 \varepsilon'/(1.8 \times 10^{10} T_* \lambda)$ $T_0 = T(0)$—a temperature on ideally heat-insulated dielectric layer surface determined by equality

$$\beta^2 T_*^{1/2} = \int\limits_{T_*}^{T_0} \left(2 \int\limits_{T'}^{T_0} F_*(T'')\,\mathrm{d}T'' \right)^{-1/2} \mathrm{d}T'. \tag{3.4}$$

4. Quantitative assay of the model

First, the temperature dependence of the loss angle tangent of polyvinyl acetate (see Fig. 1), assuming $T_* = 50\,^\circ\mathrm{C}$, will be used for quantitative assay of the model. In Fig. 3, the curves *1, 2,* and *3* are the graphs of the functions $F_*(T)$ at frequency f_0 of the AC voltage 50 Hz, 1 kHz, and 10 kHz, respectively, extrapolated to a temperature of 105 °C in comparison with the graphs in Fig. 1. The curve *1* is represented by the graph constructed by formula (3.4) of the dependence of the parameter β on the temperature T_0 on the ideally insulated surface of the dielectric layer (curve *4*). This dependence reaches the maximum value $\beta_1^* \approx 6.79$ for $T_0^* \approx 85.6\,^\circ\mathrm{C}$. The value T_0^* in this case is extremely attainable, since the coordinates of the points on the descending branch of the curve *4* determine the temperature distributions $T(z)$ in the dielectric layer that are not realizable as steady-state solutions of Eq. (3.1) with the boundary conditions (3.2). For a fixed value $\beta < \beta^*$, the value T_0 for the realizable steady-state temperature distribution is determined by the abscissa of the point on the ascending branch of the curve *4*.

In the case of $\beta > \beta^*$, there is no solution for the problem (3.1), (3.2), e.g., there appears the thermal breakdown state of the dielectric layer. Therefore, the value

$$\Delta U^* = \left(\frac{1.8 \times 10^{10} \beta^* T_* \lambda}{f_0 \varepsilon'} \right)^{1/2} \approx 1.34 \times 10^5 \left(\frac{\beta^* T_1 \lambda}{f_0 \varepsilon'} \right)^{1/2}, \tag{4.1}$$

usually called the breakdown voltage, can be used as a limiting voltage, with which it is advisable to compare the operating voltage and determine the safety factor at possible overloads. If for polyvinyl acetate we assume that $\lambda = 0.16$ Vt/(m) and $\varepsilon' = 3$ [5], then from the formula (4.1) with $f_0 = 50$ Hz follows $\Delta U_1^* \approx 80.6$ kV.

At $f_0 = 1$ kHz, the dependence of β on T_0 reaches the maximum value $\beta_2^* \approx 7.17$ for $T_0^* \approx 100.7\,^\circ\mathrm{C}$ (the curve *5* in Fig. 3), which, according to the formula (4.1), determines the breakdown voltage $\Delta U_2^* \approx$ 18.5 kV. For the frequency 1 kHz, the area under the curve *2* is much larger than the area under the curve *1* corresponding to the frequency 50 Hz. This leads to a higher level of the dielectric loss at a frequency

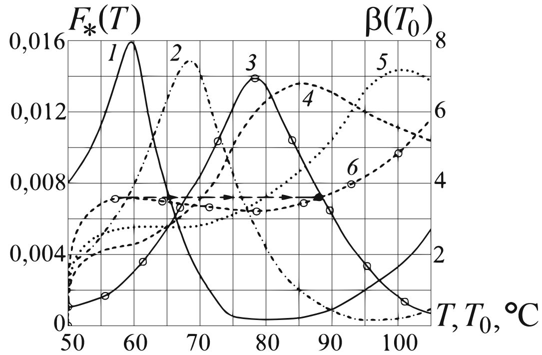

Fig. 3. The graphs of the function $F_*(T)$ for the frequency f_0: 1 — 50 Hz; 2 — 1 kHz; 3 — 10 kHz and the dependence of the parameter β on the temperature T_0 for the frequency f_0: 5 — 50 Hz; 6 — 1 kHz; 7 — 10 kHz

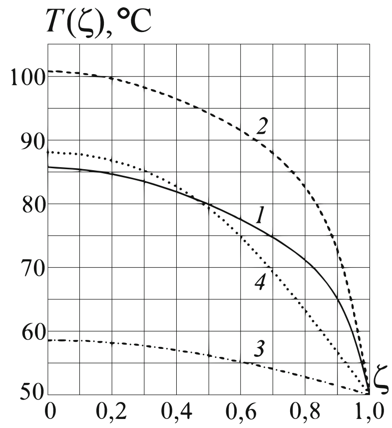

FIG. 4. The temperature distribution in the dielectric layer (polyvinyl acetate) for various values of $T(0)$: 1 — $85,6\,^\circ$ C; 2 — $100,7\,^\circ$ C; 3 — $58,6\,^\circ$ C; 4 — $88\,^\circ$ C

of 1 kHz and a more gentle character of the ascending branch of the β dependence on T_0 graph in the temperature interval 60–80 °C. Even greater is the area under the curve *3* for the frequency 10 kHz, which is the cause of the local maximum of the dependence of β on T_0 (the curve *6*) for $T_0' \approx 58.6\,^\circ$C with the value $\beta_3' \approx 3.58$. The temperature distribution in the dielectric layer, determined by the coordinates of this maximum, is unstable and under small perturbations goes into a stable state, which corresponds to the point with the dark circle with the same ordinate and abscissa $T_0'' \approx 88\,^\circ$C (the transition is conditionally represented by a horizontal dashed line with arrows). The absence of data on the dielectric properties of polyvinyl acetate at temperatures above 100 °C does not allow us to trace the ascending branch of this dependence to the maximum corresponding to the thermal breakdown state. For frequencies 50 Hz and 1 kHz, the possible increase in the values of the function $F_*(T)$ for $T > 100\,^\circ$C can also cause a nonmonotonic change in the parameter β with the achievement of additional extremum as the value T_0 increases. To clarify this assumption and to determine whether such extremum of β depends on T_0, one must have information about the dielectric properties of polyvinyl acetate at higher temperatures.

In Fig. 4, according to the formula (3.3) for the temperature distribution $T(\zeta)$ in the dielectric layer, the curves *1* and *2* are constructed for the abscissas $T_0^* \approx 85.6\,^\circ$C and $T_0^* \approx 100.7\,^\circ$C, which are the maxima of the dependence β on T_0 at frequency 50 Hz (the curve *4* in Fig. 3) and 1 kHz (the curve *5* in Fig. 3). The curves *3* and *4* in Fig. 4 illustrate the change in the steady-state temperature distribution over the thickness of the dielectric layer at $f_0 = 10\,$kHz in the case when the parameter β reaches the local maximum value $\beta_3' \approx 3.58$ (see Fig. 3, the curve *6*).

The graph of the function $F_*(T)$ for an acyclic polyimide represented in Fig. 2 makes it possible to increase the temperature range in which we can carry out a quantitative assay of the model under study. In order to take into account the influence of this dielectric properties at negative centigrade temperatures, let us use the Kelvin temperature scale and take up $T_* = 193$ K $\approx -80\,^\circ$C. In Fig. 5, the curve *1* is the graph of the function $F_*(T)$ for the acyclic polyimide in this scale. The dependence of the parameter β on T_0 (curve *2*) in this case has three maxima. The temperature distribution along the thickness of the dielectric layer corresponding to the first maximum with the coordinates $T_0' \approx 289.7$ K and $\beta' \approx 6.14$ is shown in Fig. 6 of the curve *1*. It is unstable and under small perturbations becomes a stable steady-state temperature distribution (the curve *2* in Fig. 6). This transition to the ascending part of the dependence β on T_0 to the point marked with a dark circle in Fig. 5 is conventionally represented by a dashed line with arrows. The temperature on the ideally insulated surface of the dielectric layer takes the value $T_0 \approx 484.7$ K, i.e., increases in comparison with the value of T_0' by 195 K.

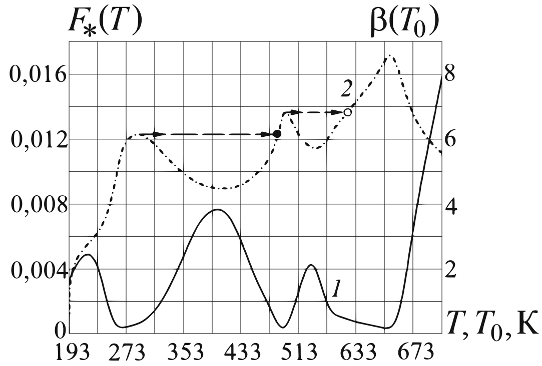

FIG. 5. The graphs of the function $F_*(T)$ (line 1) and the dependence of the parameter β on the temperature T_0 (line 2)

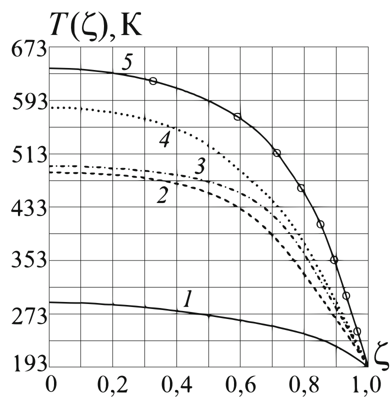

FIG. 6. The temperature distribution in the dielectric layer (alicyclic polyimide) for various values of $T(0)$: 1 — 289, 7° C; 2 — 484, 7° C; 3 — 494, 6° C; 4 — 582, 7° C; 5 — 641° C

The stable temperature distribution of the dielectric layer with the largest value of temperature $T_0 \approx 582$ K (abscissa of the point marked with the bright circle in Fig. 5 and the ordinate $T(0)$ of the curve *4* by Fig. 6) corresponds to the unstable temperature distribution (curve *3* in Fig. 6), which is in accord with the second maximum of β from T_0 with coordinates $T_0'' \approx 494.6$ K and $\beta' \approx 6.82$ (the curve *2* in Fig. 5). The temperature distribution corresponding to the third maximum of the dependence β on T_0 with the coordinates $T_0^* \approx 641$ K and $\beta^* \approx 8.57$ (the curve *2* in Fig. 5) is shown in Fig. 6, the curve *5*. The abscissa of this maximum lies near the upper boundary of the operating temperature range of the dielectric under study. Using the value ε^* for $\varepsilon' = 2$ and $\lambda = 0.35$ W/(m K) and according to the formula (4.1), it is possible to calculate the breakdown voltage $\Delta U^* \approx 23.2$ kV. One of the factors ensuring a sufficiently large value ΔU^* in this case is relatively high for organic polymers thermal conduction coefficient of alicyclic polyimide.

5. Conclusion

The thermal breakdown mathematical model of a plane layer of a polar dielectric having a nonmonotonic temperature dependence of the dielectric loss is constructed. The model is based on the integral relation, allowing us to estimate the breakdown voltage and the parameters determining the temperature distribution in the layer preceding the occurrence of thermal breakdown. The study gives the quantitative assay of the model for the two types of polymer materials used as dielectrics.

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